Complex Envelope Soliton in Bose-Einstein Condensate with Time Dependent Scattering Length

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We elaborate on a general method to find complex envelope solitons in a cigar shaped Bose-Einstein condensate in a trap. The procedure incorporates time dependent scattering length, oscillator frequency and loss/gain. A variety of time dependencies of the above parameters, akin to the ones occurring in the experiments can be tackled.

Coherent atom optics is a subject of much current interest for its relevance to both basic physics and technology. For this purpose, lower dimensional condensates *e.g.*, cigar shaped Bose-Einstein condensates (BECs) have been the subject of active study in recent years [1]. Intense investigations about the behavior of the condensate in the presence of time varying control parameters like, scattering length, oscillator frequency *etc.*, are being carried out for the purpose of optimal control. In BEC, solitary waves have been experimentally observed, both in repulsive and attractive domains [2, 3]. These have been understood from the non-linear Gross-Pitaevskii (GP) equation describing weakly coupled BEC,

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = \left\{-\frac{\hbar^2}{2m}\nabla^2 + U|\Psi(\mathbf{r},t)|^2 + V\right\}\Psi(\mathbf{r},t), \tag{1}$$

where $U = 4\pi\hbar^2 a_s(t)/m$, $a_s =$ the s-wave scattering length, and V is the external potential. The scattering length, which is the coefficient of nonlinearity in GP equation, can be adjusted both in sign and magnitude through Feshbach resonance. Recently a family of exact solutions of quasi-one dimensional GP equation with time varying scattering length, loss/gain, in the presence of oscillator has been obtained [4]. These are chirped solitons with real envelope. In recent years, solitons with complex envelope is generating interest in different areas of physics. These are known as Bloch solitons in condensed matter physics. In optical fibers one also finds these solitons. In BEC, complex solitons have been investigated [5, 6, 7, 8, 9]. Very recently complex envelope solitons in optical lattices has been reported [10].

In the present study, we explicate a method to obtain general complex envelope solitons in cigar shaped BEC, in a trap incorporating time dependent scattering length, oscillator frequency, and loss/gain. We have constructed the two hydrodynamic equations in this scenario and have found their solutions. Our analysis starts with quasi-one dimensional non-linear Schrödinger equation (NLSE) derived from three dimensional GP equation with an additional loss/gain term $g(t)$ and time dependent chemical potential $\nu(t)$. In dimensionless units this is given by [4],

$$i\partial_t\psi = -\frac{1}{2}\partial_{zz}\psi + \frac{1}{2}M(t)z^2\psi + \gamma(t)|\psi|^2\psi + \frac{ig(t)}{2}\psi + \frac{1}{2}\nu(t)\psi. \tag{2}$$

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We assume the following ansatz solution

$$\psi(z,t) \;=\; \sqrt{A(t)}F\left[A(t)z - l(t)\right]\exp\left[i\{\chi(z,t)+\phi(z,t)\}+\frac{G(t)}{2}\right], \qquad (3)$$

where $\sqrt{A(t)}F[A(t)z - l(t)]\exp[i\chi(z,t)]$ carries the signature of complex envelope. Further $G(t)$ and $l(t)$ have been defined as,

$$G(t) \;=\; \int_0^t g(t')dt' \;,\;\; l(t) = \int_0^t v(t')dt'. \qquad (4)$$

The phase is chirped as,

$$\phi(z,t) \;=\; a(t) + b(t)z - \frac{1}{2}c(t)z^2 \;, \text{ where, } \;\; a(t) = a_0 - \frac{1}{2}\int_0^t A^2(t')dt'.$$

Substitution of the above ansatz in Eq.(2) allows one to separate this equation in imaginary and real parts. The knowledge of the earlier conditions, leads one to write the imaginary equation as,

$$\begin{aligned} -uF' &= -F'\chi' - \frac{1}{2}F\chi'', \text{ and} \\ \chi' &= u - \frac{2C_0}{F^2}. \end{aligned} \qquad (5)$$

Here prime denotes derivative with respect to T, where $T = A(t)[z - l(t)]$. We have set $\frac{l_t + cl - A}{A} = u$, $A_t = Ac$ and C_0 is an integration constant. Redefining $C_0 = \frac{uF_0^2}{2}$, $F = \sqrt{\sigma}$ and $F_0 = \sqrt{\sigma_0}$, Eq.(5) leads to,

$$\chi' = u\left(1 - \frac{\sigma_0}{\sigma}\right). \qquad (6)$$

We have imposed the boundary condition, $\chi' \to 0$ for $\sigma \to \sigma_0$. The local phase velocity of the solitary pulse is described by χ' and local density is described by σ, σ_0 represents equilibrium density.

From the real equation one can extract a Riccati type equation, $c_t - c^2(t) = M(t)$ which yields the solution of $c(t)$. The real equation also yields,

$$F'' - \mu F - 2\kappa F^3 \;=\; \chi'^2 F - 2u\chi' F, \qquad (7)$$

where $\kappa = \frac{\gamma_0}{A_0}$. Applying the following consistency conditions, one obtains

$$\begin{aligned} \gamma(t) &= \gamma_0 e^{-G}A(t)/A_0 \;,\; b(t) = A(t), \\ A(t) &= A_0 \exp\left[\int_0^t c(t')dt'\right], \; \nu(t) = A^2\frac{\mu}{2}, \end{aligned}$$

and,

$$F'' - \epsilon F - 2\kappa F^3 - \frac{\lambda}{F^3} = 0, \qquad (8)$$

where $\epsilon = \mu - u^2$ and $\lambda = u^2 F_0^4$. Integration of Eq.(8) leads to the convenient form,

$$\left(\frac{\partial\sqrt{\sigma}}{\partial T}\right)^2 \;=\; (\kappa\sigma - u^2)\frac{(\sigma-\sigma_0)^2}{\sigma}. \qquad (9)$$

The solution for Eq.(9) is,

$$\sigma(z,t) = \sigma_0 \left\{ 1 - \cos^2\theta \ \mathrm{sech}^2 \left[\frac{A(z-l)\cos\theta}{\zeta} \right] \right\}. \tag{10}$$

These are the dark and grey solitons in the distributed scenario.

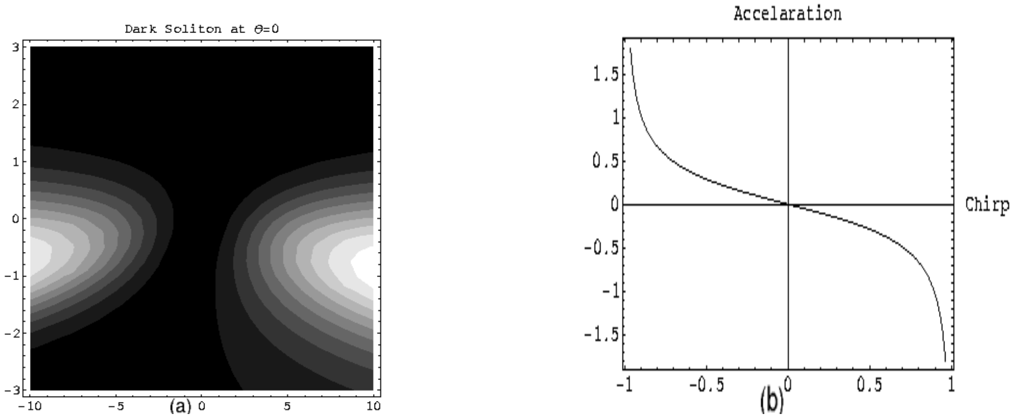

Figure 1: (a) Density profile of the Condensate and (b) Chirp Controlled Soliton Motion

In conclusion, for quasi-one dimensional GP equation in an oscillator potential with time dependent coupling and loss/gain, a general complex dark soliton profile is obtained. In the presence of expulsive oscillator the motion of the condensate is controlled by the sign of the chirp, *i.e*, whether the soliton is accelerating or decelerating.

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